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OPERATIONAL DESIGN CRITERIA FOR MISSILE GROUND SYSTEMS: READINESS TESTING

Sidney I. Firstman
Engineering Division
The RAND Corporation

P-2112

September 15, 1960

To be presented at the 18th National Meeting of the Operations Research Society of America, October 10-12, 1960, Detroit, Michigan.

ABSTRACT

Parts of a missile perform physical roles--they propel, guide, etc. Additionally, they play operational roles--their joint operation must accomplish a mission, and because missile hardware is imperfect, i.e., has failure characteristics, it necessitates certain ground operations. Readiness testing, one of these ground operations, is that set of tests performed regularly for the purpose of detecting failures that have occurred in the missile and launch equipment. An important adjunct to criteria for physical characteristics of readiness-testing equipment are statements derived from operational analyses that specify (a) what tests are best done by each testing method, (b) the best test frequencies, and (c) the preferred equipment locations for each test; these are operational design criteria. Different testing methods--check periodically, monitor continuously, leave alone--and equipment locations--van, silo, missile--could yield different readiness probabilities for each missile function. These probabilities depend on the particular missile's characteristics (test-point availability, failure rates, modes of operation, etc.) and test equipment characteristics (test completeness, accuracy, failure rates, etc.). In addition, missile system concepts or constraints (fixed period for inspections, required tests, limited weight on missiles, etc.) must be accounted for in the determination of the best readiness-testing method and test equipment location for each function. This is done in an analysis employing Markov Processes to describe the system operation and a discrete programming formulation for the decision process. Only the essence of the analysis is described. It is an example of the application of Operations Research to problems of missile ground system design. Introductory comments include a discussion of the

roles of Operations Research in equipment design, and a discussion of the general readiness-testing design problem.

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INTRODUCTION

Criteria for the design of a missile's ground system are of several natures. At present, typical criteria will state conditions like, "electronic equipment must be capable of unattended operation for a period of 60 days," or "electronic and hydraulic equipments will be designed so that they may be maintained using a remove and replace concept." In conjunction with these criteria for the physical requirements of the equipment, statements derived from objective studies are needed to specify what the equipment is to do, how well it must do these tasks, how often, and so forth. These are operational design criteria; they are rules for the use of the equipment, and goals or guides for the design of the equipment. In the past, in contrast to the analytical work done to define the roles and content of the missile, little objective attention has been given to such criteria. Instead, they have been set by the methods of "rule of thumb" and "engineering judgment." This need not be the case, however, for the mathematical methods that we typically associate with the field of Operations Research--the same methods that have been useful for determining operating rules for systems--can be employed to determine such operational design criteria.

The utility of an OR study in the realm of design is limited, certainly; a design cannot be completed by an OR analysis. An OR study can, however,

examine the requirements placed on the system, and in some cases can determine equipment design goals that tend to optimize the integrated system design or performance, and that can serve as points of departure for physical design studies. This paper presents the essence of such an analysis.* The paper examines the readiness-testing requirements for a fixed-site ballistic missile and describes (a) a mathematical model for estimating the dependence of missile readiness on relevant physical and operational design factors, and (b) an aid to the determination of a readiness-testing program (i.e., operational design criteria) for each missile function.

Operations Research and Equipment Design

In addition to their usefulness in determining design goals within which detailed design decisions can be made, operational analyses have made contributions of at least two other natures to equipment design. It is sometimes true that while an engineer with a particular design forte will apply his best objective and quantitative design knowledge and efforts to design problems in his field, he will tend to use subjective arguments when asked to specify the total system design. Often such a designer will either not consciously consider the problems of optimizing other than within each specific design, or he will minimize, in his mind, the feasibility or importance of the over-all design optimization. This is especially true if the over-all design requires design disciplines other than his own. For some of these system design problems, the tools of OR are well suited to optimize the over-all design, and this leads to the second area of possible contribution of OR to equipment or system design. Some design problems can

*The details of the analysis can be found in Ref. 1.

be formulated and solved using mathematical techniques that are relatively new, and, for the most part, unique to the field of OR.

The technical literature contains numerous examples of systems that are not optimally designed while elements of the system are so designed; this is a design-marriage problem. For some of these cases, an OR study can show ways to optimize the system design by calling for the best combination of element designs. One exaggerated example of this design-marriage problem is discussed in a paper by Bailey.[2] In that article the design of an aircraft is shown as envisioned by each of the separate design-groups--engine, armament, production, etc. The aircraft as seen by the engine design group has an airframe that is drawfed by the engine; the armament group sees the aircraft much like a flying battleship; while the production groups' ideal design is an aircraft constructed of straight boards, nailed together. The article uses this design comedy to illustrate the role of OR in design integration for optimum system performance.

It follows then that an OR analysis will often be concerned with optimizing those parameters of a system that are typically determined by subjective reasoning. For a successful analysis (as done in other areas of OR application), these parameters will be quantified, as will be the concepts, notions, and reasons that would be employed for the subjective analysis. Here, then, is the third area of possible contribution of OR to equipment or system design; an OR analysis can quantify and make explicit some of the notions, reasons, concepts, and parameters that heretofore have been expressed qualitatively and employed subjectively. A brief example may help make this notion explicit.

When investigating the pre-launch checkout problem reported in Ref. 3,

the "value" of a check was derived. This "value" pulled together into a quantifiable term, the notions of (a) the chance of failures existing prior to the test, (b) the capability to find failures, and (c) the chance of causing failures during the test. Persons concerned with designing check-out equipment knew that there were some relationships among these factors that were important, and this "value" term made one such relationship explicit.

This is an example of important (from a system operation viewpoint) design concepts that are vaguely defined by intuition, common sense, and subjective arguments, but that can be defined in a specific and quantitative fashion as the result of an OR analysis. This quantification is important and useful to design, in its own rights, in much the same manner that a good vocabulary is useful to speech and thought. Without proper words, thoughts can be conceived and expressed only imperfectly; without explicit statements of design issues, the optimization cannot be complete. When employed in proper context, then, an OR analysis can serve as an aid to design intuition and thinking.

Certainly there are shortcomings to the operational design analyses that are performed. As for other areas of application of OR, operational design is hampered with a shortage of good data. Reliability estimates or performance productions must often be used, and these are often unreliable. Emphasis must be placed, then, on developing design decision methods that are insensitive to large data errors. Planning and decision methods are needed that are useful in the face of poor data, and that would give useful design rules that accommodate uncertainty in such factors as reliability or performance.

READINESS TESTING DESIGN PROBLEM

Parts of a missile (missile functions) perform physical roles--they propel the missile, they guide the missile, etc. In addition, these functions play operational roles--they must operate as a unit to accomplish a mission, and because missile hardware is imperfect, i.e., it demonstrates failure characteristics, it necessitates certain ground operations. Readiness testing is one of these ground operations that is made necessary by missile unreliability, and its effectiveness is measured in missiles ready, which is an operational term. Within the limits imposed by the system cost and operational concept, a missile should be ready to launch as much of the time as is possible.

Readiness testing is that set of tests performed prior to alert, according to a test program, for the (primary) purpose of detecting failures that have occurred in the missile and launch equipment, so that these failures can be corrected.

Numerous physical, operational and cost factors enter into an investigation of readiness testing. Failures in the missile can be detected only by testing, but failures caused by testing also act to degrade the missile's readiness. Consequently, the possibility of these test-caused failures must be accounted for in the development of a readiness-testing program.

Among the other factors that enter into the development of a readiness testing program are the potential capabilities of the test equipment to detect and isolate faults, and the expected propensities of the test equipment to falsely indicate a defective condition. Missile failures that are undetected will, potentially, degrade the effectiveness of the force, and good parts that are called bad by the test equipment result in unnecessary

down time and expense for repair.

Failures in the missile and the test equipment characteristics are the physical factors that enter into the development of a readiness-testing program; they are reflected in the generation of test and repair requirements. In addition, the resulting operating times, such as the time required to test a missile and repair malfunctioned parts, must be accounted for. It is possible that the system maintenance concept includes a planned delay in repairing a known malfunction; this could prove economical and must also be accounted for. The reasons for these requirements to account for test, repair and maintenance times are simply that a missile must be ready to launch in order to be launched, and if a missile is undergoing test, maintenance, or awaiting maintenance, it is certainly not launch ready.

Lastly, there are costs associated with missile testing and repairs, and when a system concept is being prepared, these costs should play a major role. Dollars spent on test and maintenance could, perhaps, be better spent for more missiles.

It can be seen then that much more than just the details of such physical factors as the test signal requirements, and measurement techniques enter into the determination of a readiness-testing concept and program. Readiness testing plays an operational role; it affects, and in turn is affected by other operations. Therefore, these operational factors should be accounted for when determining such concepts and programs. An operational analysis would be integrated with hardware analyses, as estimates based on such physically oriented studies are necessary inputs to an operational study. Then the operational study would define design goals that tend to give the best operational capability, and these goals must in turn be translated into physical items.

System Readiness-Testing Concept

It appears that a complete readiness-testing concept for a missile is developed in two steps. First, a readiness-testing policy or concept must be developed for the missile as a whole, operating within the missile system; this would be integrated into the system design concept. Following this determination, the readiness-testing method for each black-box or function within the missile and launch equipment must be determined; this would be a more detailed analysis done within the framework of the system concept. The criteria by which preferred methods and frequency of testing are chosen need not be the same for both levels, and particular attention must be paid to the proper choice of criteria.

A missile system, by the nature of its design and anticipated employment, will tend to dictate a broad concept (or at least limits) for its readiness testing. (For example, a missile that is to have an in-silo repair capability could probably be checked periodically with little or no additional cost for checkout equipment, as maintenance testing equipment could also serve as peacetime confidence testing equipment.) Within the broad context dictated by the missile operational and physical characteristics, other considerations would then be brought to bear to determine the system readiness-testing concept; system cost is the primary consideration in this realm.

When determining the system operational and design concepts, one should be highly concerned with the system cost. Each element of the system design and operation should be evaluated against the dollar sign, and the system readiness testing concept, i.e., whether it be continuous monitor, periodic check, or leave alone, should be chosen with the dollar sign a critical decision factor.

The rationale for this cost criticality at this stage of design is that, ideally at least, support dollars can be exchanged for missile dollars when making a system proposal or evaluation. Missile systems are often compared on a cost-effectiveness basis, and in this type of evaluation one important effectiveness determinant is the number of missiles launched for a given system cost. Missiles launched per system dollar are determined by how many are procured, how they are based, and how they are supported. At this stage it is relatively straightforward and certainly meaningful to evaluate alternative support concepts on how they contribute to the over-all system effectiveness when their costs are traded-off with the alternative uses for the same dollars.

Once the system support concept has been determined along with the system operations and basing concepts, the cost constraint often loses its meaning; a cost context then becomes more appropriate. For the following stages of design it is more important to match the physical designs of the missiles, facilities and ground equipment, and to develop the methods for support for the system, within the system concept, to complement the physical designs of the missile and facilities.

An important element in the investigation of a system concept is that system characteristics must be employed. For example, the determination of a system periodic inspection policy must be based on a complete missile mean-time-to-failure, and any failure within the missile must be considered in the same manner. Average repair times and average repair costs are elements of an investigation on the system level.

<u>Readiness Testing of Functions</u>

On the blackbox or function level, attention should be directed more

closely toward adapting the ground system and the flight article to each other. Within the system concept, physical characteristics of the missile and its required operations can be employed to mold a test program and ground equipment to the missile. Within the same examination, the possibility of modifying the initial design of the missile to improve the over-all system capability should not be overlooked. The total missile-ground system must function for the missile to be effective and therefore the missile- and ground-systems designs should be complementary.

Much good work has been done on the problem of determining preferred system testing concepts, see for examples Refs. 4, 5, and 6. The balance of this paper will be devoted to the complementary problem of developing a test program for each function within a previously chosen system concept.

A missile emplaced in a silo is tested in several ways, using equipment in several locations. (See Fig. 1.) For safety and physical reasons some functions on the missile are monitored continuously; two examples are propellant or oxidizer tank pressure and gyro fluid temperature. Other functions are essentially left alone for extended periods of time; storable (and solid) propellant engines are a case in point. Some functions, and indeed the major portion for most missiles, are checked periodically. They are checked because failures do occur while a missile is in a static mode, and these failures should be detected and repaired before a launch attempt is to be made.

Once the entire system-design-and operation concept has been determined, the test equipment must be designed to operate within the concept which will set certain bounds on the design, e.g., the time between periodic inspections and replacements. One important question for the equipment designer is, for

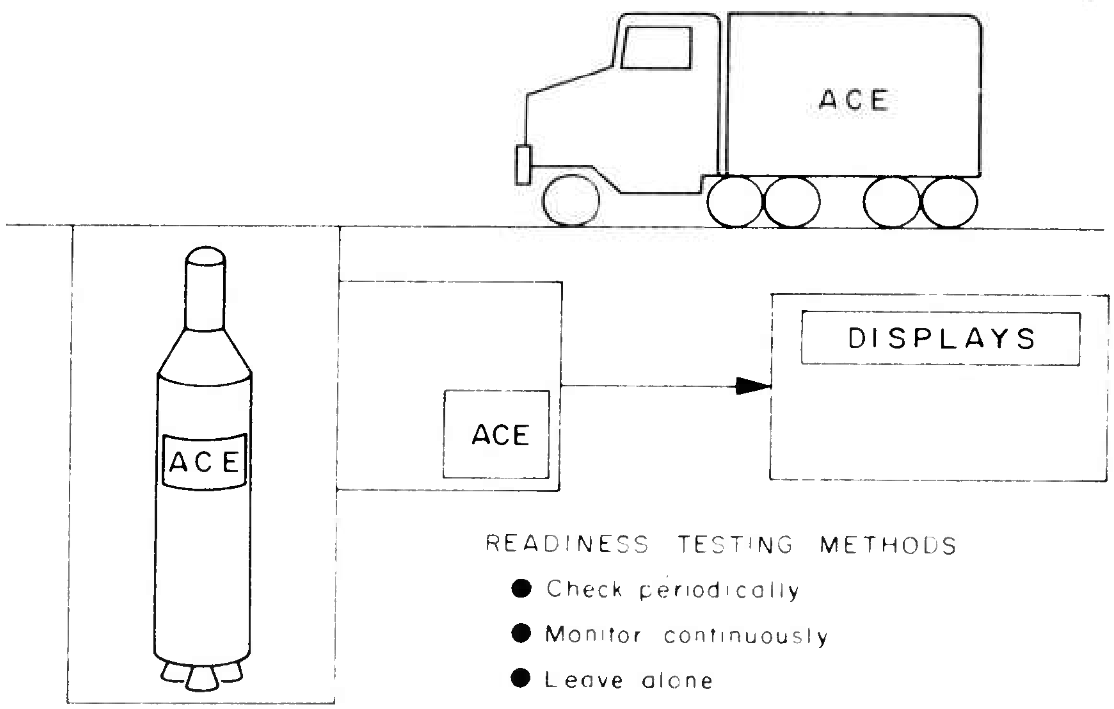

Fig. 1 — Location of test equipment

those missile functions for which design freedom exists, how does one decide whether to check it periodically, or to monitor it continuously (if this is possible), or, perhaps, to leave it alone until a launch attempt is to be made? Answers to these questions determine the nature of the test equipment to be employed within the system concept, and the following mathematical model and decision methods are intended to aid engineers in such design decisions.

MATHEMATICAL DEVELOPMENT

Briefly, the readiness-testing equipment design problem we are addressing is as follows (see Fig. 1). Test equipment can be located in a van, in the silo, or aboard the missile. For some systems a helicopter might replace a van. The test equipment in all three places could be used to check periodically, while the silo and missile equipment could also be used to monitor continuously. Because of their different capabilities to detect failures, the timeliness of their detections, and their varying propensities toward causing failures, each testing method will (in general) give a different readiness for each missile function. This readiness is measured by the probability that the function will be operative at a randomly chosen future time, e.g., the time when the missile must be launched.

In order for a missile to be ready to go at any instant, each function must also be ready to go at that time. This characteristic for the missile and launch equipment will be called P(up); i.e., the probability that the missile is ready-to-go (is up) at any time chosen at random. The missile P(up) is dependent on the readiness probability of each missile function.

The problem to be solved then is: what test actions should be taken to maximize the missile P(up) within the constraints imposed by the missile

design, system operation, safety, and other relevant conditions. Restated in other terms, within the confines of the system concept, how does one choose among the choices of checking periodically, monitoring continuously, or leaving alone, and among the possible locations for the test equipment, for each function of a missile and launch equipment, in order to make best use of physically constrained tests and obtain the greatest P(up).

The first step in the analysis is to describe a missile function's life pattern in terms of movements through states. Then the function's readiness probability will be determined in terms of state-to-state transition probabilities. Each of these transition probabilities must be expressed in terms that describe the physical and operational elements of the system, and that can be measured or estimated with adequate accuracy. (The results of the analysis are essentially insensitive to uncertainty in some parameters.) Following that, a means for choosing among test methods for each function must be developed. Actually, two means for choosing test methods must be developed; first, one is needed for cases when no real constraints are operative other than those imposed by the system concept and safety and physical considerations; then, a method is needed for dealing with other constraints, such as space and weight limitations for the test equipment. Only the first problem will be considered in this paper; the second problem is discussed in Ref. 1. For convenience, the analysis will be described in a partially reverse order.

Problem Statement

A missile and ground-operating equipment necessary for launch are assumed to be composed of N statistically independent parts or functions. It is further assumed that the state--good or no good--of each of these

functions can be determined in terms of function performance necessary to insure proper system performance. Because it forces limits for a decision of good or bad to be made in an a priori sense, this assumption neglects the chance that while a function parameter may have drifted beyond an a priori set limit, other system changes may have occurred to compensate for this drift. This undesirable feature can be alleviated by aggregating missile parts into larger test units.

It follows from the assumption of statistical independence* that P(up) for the entire weapon, noted simply by P, is given by

$$P = \prod_{i,j,k} (P_{ijk})^{x_{ijk}} \qquad (1)$$

where

P_{ijk} = P (function i is operative, i.e., in ready condition, if tested using method j and equipment in location k), and

$$x_{ijk} = \begin{cases} 1, & \text{if readiness-testing method } j \text{ and equipment in location } k \text{ is used for function } i \\ 0, & \text{otherwise} \end{cases}$$

To separate those functions that are constrained by safety or physical reasons from those for which design freedom still exists, let the constrained

*This assumption of statistical independence does not hold strictly for all test methods. For example, using the continuous monitor method a common test item may monitor several functions. If these functions are not grouped, then the P_{ijk} estimate for each will reflect the test equipment's failure probability, and hence, P given by Eq. 1 will be a conservative estimate. When this independence assumption fails, the effects on P could be significant, but the decision process to be described will be largely unaffected. This is because each P_{ijk} is compared to each alternative P_{ijk} estimate on an individual basis and in this context each estimate is, in essence, separate of the other estimates (except, of course, that all terms are based on this same system concept) because at that time, just it is being examined with all other factors being held constant.

test be indicated by i_c. Then

$$P = \prod_{\substack{i=i_c \\ j,k}} (P_{ijk})^{x_{ijk}} \prod_{\substack{i\neq i_c \\ j,k}} (P_{ijk})^{x_{ijk}} \tag{2}$$

The design problem can now be stated as:

Within the constraints imposed by safety and physical reasons, and within the system readiness testing concept, determine that set of $x_{ijk} = 1$ so that P given by Eq. (2) is maximum and

$$\sum_{j,k} x_{ijk} = 1, \text{ for each } i. \tag{3}$$

The method of solving this problem is nothing more than a search for the best P_{ijk} for each unconstrained function. A design tableau is useful to facilitate this search and to afford designers the opportunity to examine alternative test groupings. First, a useful design tableau will be presented; then, the method of estimating the P_{ijk} terms will be considered.

Design Tableau

The readiness probability estimates for each function-test method-equipment location combination could be presented on a tableau . Then, for each of the functions with design freedom, the test method that affords the largest readiness probability for each function could be identified and selected. Physical considerations, however, could cause a compromise of this selection to be desirable. For example, some tests can be done with common equipment and the above decision process does not take account of this opportunity for economy. If one test that were best done in a

continuous monitor manner could be done using the same equipment as another test best done in a periodic manner, then it could prove economically or operationally desirable to do both tests periodically. If the readiness were organized into a tableau such as Fig. 2., then the effects on the over-all P(up) of such compromises or combinations could be readily observed.

In this tableau the functions 1, 2, 3, ..., represent single functions or groups of functions that are always best tested as a group. The functions m and n show the possibility for different groupings depending on the test method and test equipment location.

To use such a tableau to aid in the design decision process, one would

1. Indicate by circling those tests that must be done for reasons of safety or physical requirements.
2. Fill in the balance of the readiness terms for each test method and equipment location combination with data from the appropriate graph (Fig. 3, p. 25, is an example).
3. Select the largest readiness probability for each unconstrained function and indicate it with a V.
4. Form the product of those P_{ijk} terms (one from each column) that are marked with a V or O; this is the "best" over-all P(up).
5. To examine other convenient test groupings, determine what tests can be grouped with other tests and enter these in the row Tests to Group.
6. Use Grouping Comparison row to compare the products of the grouped terms with the products of the "best"

EQUIPMENT LOCATION	TEST METHOD	FUNCTION OR GROUP OF FUNCTIONS											
		1	2	3	• • •	m_1	m_2	m_3	• • •	n_1	n_2	n_3	• • •
VAN	CP*	p_1	p_2	p_3		p_{m_1}	p_{m_2} p_m	p_{m_3}					
SILO	CP												
	CM									p_{n_1}	p_{n_2} p_n	p_{n_3}	
MISSILE	CP												
	CM										p_n		
NONE	LA	p ✓											
CHECK IF DONE		✓		✓							✓		
P (UP) CHOSEN													
TESTS TO GROUP CP CM													
GROUPING COMPARISON													

CP*—CHECK PERIODICALLY
CM —CONTINUOUSLY MONITOR

LA—LEAVE ALONE
✓—BEST CHOICE OF P (UP)

○—CONSTRAINED BY SAFETY OR PHYSICAL CONSIDERATIONS

Fig. 2 — Unconstrained design tableau

P(up) terms, and determine the worth of non-best test patterns.

The P_{ijk} terms must now be quantified in terms describing germane physical and operational factors. These terms should be readily estimable to be of use.

Derivation of P_{ijk} Terms

Under the assumption that a missile function is either good or not-good at any time, the portion of the life span of a function that is spent in the silo can be divided into five states:

1. Operative, there are no failures (malfunctions) in the function.
2. Inoperative--unknown; a disabling failure has occurred in the function so that it is inoperative but the failure is undiscovered.
3. Inoperative--awaiting maintenance; a disabling failure has occurred in the function, it has been discovered, and the missile is down awaiting maintenance.
4. Being maintained; the inoperative function is being repaired or replaced while the missile is down.
5. Undergoing Periodic Inspection or Preventive Maintenance; the missile is down while the function (or all functions) are being inspected or replaced.

Under these assumptions and definitions, it is necessary that all functions on a missile be in state 1, i.e., operative, in order for the missile to be ready to go. It is plausible that preventive maintenance could be performed after a missile is removed from the silo and replaced with another. In

this case, the down time is counted against the maintenance operation and should be considered when developing the system maintenance concept, but is not germane to this analysis of readiness testing.

Looking now at each function separately, it is seen that its life is a series of states with defined paths of possible transitions from state to state. If the missile is rejuvenated by preventive maintenance every several years (perhaps this period is determined by the shelf life of storable propellants or composition seals) then it appears that wear-out failures can be safely neglected in an examination of readiness testing, except that proper attention must be given to the constraints imposed on the allowable types of tests for some functions.* For example, mechanical parts with definite wear-out characteristics probably should not be continuously exercised. For the readiness-test concept, the concern should be centered about the randomly occurring failures. And, for this analysis, one should also take account of the failures that are caused by the readiness testing; the different methods of testing impose different stresses on the missile functions.

One further assumption is needed before the analysis can continue; that of exponentially distributed failures. The justification for this assumption and the conditions under which it is reasonable have been discussed in numerous reports (Refs. 4, 7, 8, and 9, for example) and will not be discussed here. This appears to be a reasonably good assumption for many physical functions, other than electronic, and there is much experimental evidence to justify its (cautious) use.

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*Wear-out phenomena should be considered when determining the preventive maintenance concept--especially for functions that are operated continuously.

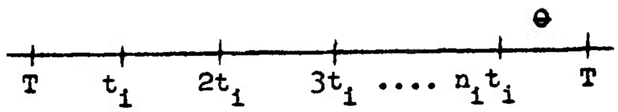
Because of the preceding assumptions on the statistics of the failure distribution, it is seen that the probability of a function failing in a given period is dependent only on the length of the period and the operational stresses, and is thereby independent of the number of periods of previous use (up to the time when preventive maintenance is necessary). All other state-to-state transitions depend only on the existing state and the state to which the movement is to be made. Therefore, the progress of a particular function through its operational lifetime can now be viewed as a Markov Process which will be described.

Check Periodically

The process of checking periodically is the most tractable of the two active readiness-testing methods and will be treated as an example; the monitor continuously and leave-alone methods are discussed in Ref. 1.* The length of time between periodic inspections that gives the best system performance is assumed to be given by the system concept; this time between periodic inspections will be called T. Each missile function that uses the check periodic method is inspected every T days, and because of the nature of the system, defects that have occurred in the function can be detected only at the time of the periodic inspection.

It will prove useful to break the T days down into n_i smaller time periods of t_i hours each and the time required for the missile periodic inspection θ so that a periodic interval appears as

*As implied earlier, the intent of this paper is to expound the requirements for operational design criteria and the applicability of Operations Research to ground system design. Details of the analysis are in Ref. 1.



To further simplify the analysis, the quantity of time t_i is defined to be the average length of time required to repair or replace the ith function. This includes all time required to travel to the missile, unbotton the silo, remove and repair or replace, etc.

Projecting the function's operations into the future, it appears as a series of intervals of length T, that is finally ended at the time of launch.

The movements of a missile function through this pattern must of necessity demonstrate periodic properties. The chance of a transition from one state to another will depend on whether the potential movement occurs during a periodic inspection, or between inspections. It will be convenient, therefore, to define three transition matrices: A defined over t_i, B defined over θ, and C defined over T.

For the states numbered:

1. Operative
2. Inoperative--unknown
3. Inoperative--awaiting maintenance
4. Being maintained

(NOTE: The state of being inspected is handled differently; a transition matrix will be defined for it.)

A $\equiv$ matrix of single-step transition probabilities in time t_i, for the

interval between periodic inspections.

$$A = \left\{a_{rs}\right\}; \text{ a matrix of terms } a_{rs} \tag{4}$$

where

$a_{rs} \equiv P$ (transition from state r to state s in one interval of length t_i).

Similarly, for the periodic inspection

$B \equiv$ matrix of single-step transition probilities in time θ, for the periodic inspection.

$$B = \left\{b_{rs}\right\} \tag{5}$$

where

$b_{rs} \equiv P$ (transition from state r to state s during the periodic inspection).

The process to be described moves through n_i steps of length t_i described by A, and one step of length θ described by B, for each large step, or period, in its life. A matrix is needed to describe transitions over the entire period T.

$C \equiv$ matrix of single-step transition probabilities in time $T = n_i t_i + \theta$.

If

$c_{rs} \equiv P$ (transition from state r to state s in one interval of length T),

and if it is observed that each term c_{rs} is a term compounded from a_{rs} and b_{rs} terms; that is (changing notation slightly for the sake of clarity), if $a_{rv} = p_{rv}(t_i)$, the probability of moving from state r to state v in time t_i and $b_{vs} = p_{vs}(\theta)$, the probability of moving from state v to state s in time θ, then

$$c_{rs} = p_{rs}(n_i t_i + \theta) \tag{6}$$

$$= \sum_v p_{rv}(n_i t_i)\, p_{vs}(\theta). \tag{7}$$

Where

$$A^{n_i} = \left\{ p_{rv}(n_i t_i) \right\},$$

and

$$B = \left\{ p_{vs}(\theta) \right\}, \tag{9}$$

this means that

$$C = \left\{ c_{rs} \right\} \tag{10}$$

is given by

$$C = A^{n_i} B. \tag{11}$$

The next step in finding the long-run probability of being in state 1 is to define the m-step absolute probability vector $x^{(m)}$.

$$x^{(m)} \equiv (x_1{}^{(m)}, x_2{}^{(m)}, x_3{}^{(m)}, x_4{}^{(m)}) \tag{12}$$

where

$x_k{}^{(m)} \equiv P$ (function being in state k after m periodic intervals)

and

$$x^{(m+1)} = x^{(m)} C \tag{13}$$

If matrix C is either regular or ergodic (Ref. 10), this vector has the property that if the process is allowed to continue sufficiently long

that the effects of the initial distribution of states have been dissipated, then for some large m, a steady state, or fixed, probability vector is given by

$$x = xC \tag{14}$$

That is, the long-run distribution of states following a periodic inspection should be unaffected by an additional period and periodic inspection.

Solving for the vector x (for this problem, this involves the solution of 5 simultaneous linear equations; one for each x_k and one that expresses the sum of the x_k's equals unity) gives the probability of the function being in each state following a periodic inspection that is conveniently long removed from the time of initial installation. In particular, this could be the inspection that precedes the beginning of hostilities.

Hostilities can begin at any randomly chosen time (with respect to the readiness testing schedule) and, therefore, a measure is needed for the probability of each function being operative at any randomly chosen future time. If the vector describing the state probabilities in the time following a (steady state) periodic inspection is given by x, as before, then for r time periods of length t_i later

$$x^{(r)} = xA^r \tag{15}$$

where

$$x^{(r)} = (x_1^{(r)}, x_2^{(r)}, x_3^{(r)}, x_4^{(r)})$$

Neglecting Θ, for $\Theta < t_i$ and in turn $t_i \ll T$, and therefore only a small error will result, the probability of the function being operative at time of launch is

$$E\left[x_1^{(r)}\right] = P_{ijk} = \frac{1}{n_1}\sum_{r=1}^{n_1} x_1^{(r)}, \quad \text{for each } j \text{ and } k. \qquad (16)$$

The reason for this form of the expectation of $x_1^{(r)}$ over the n_1 time interval is because the launch attempt could occur at any time with equal likelihood.

The necessary matrices must now be developed using terms describing the missile function's physical characteristics, the test equipment's capabilities and error propensities, and the system operation and maintenance concepts. This is an involved and somewhat lengthy process, but not difficult. Because it is not essential to an understanding of the essence of the problem and solution, it will not be presented in this paper; it is contained in Ref. 1. The results of such an analysis are curves showing the dependence of P_{ijk} on the several parameters; Fig. 3 is a plot for one set of parameters for the check-periodically test method. Such curves for the continuous monitor and leave-alone methods can be obtained by similar analyses.

It is seen that the descriptions on Fig. 3 are based on such factors as 1) the function reliability, expressed in mean-time-to-failure under ground conditions; 2) the test equipment access and accuracy which give the p and q estimates which are, respectively, the probability of the tester calling a good function good, and the probability of the tester calling a bad function bad; 3) the test period T, obtained from the system concept; 4) the chance of not causing a failure by testing d; and 5) a measure of the maintenance concept a, i.e., an accounting for planned delays in repairing a known failure. For these particular curves, the average time to repair a malfunctioned part, t, was included in the analysis, but its impact on

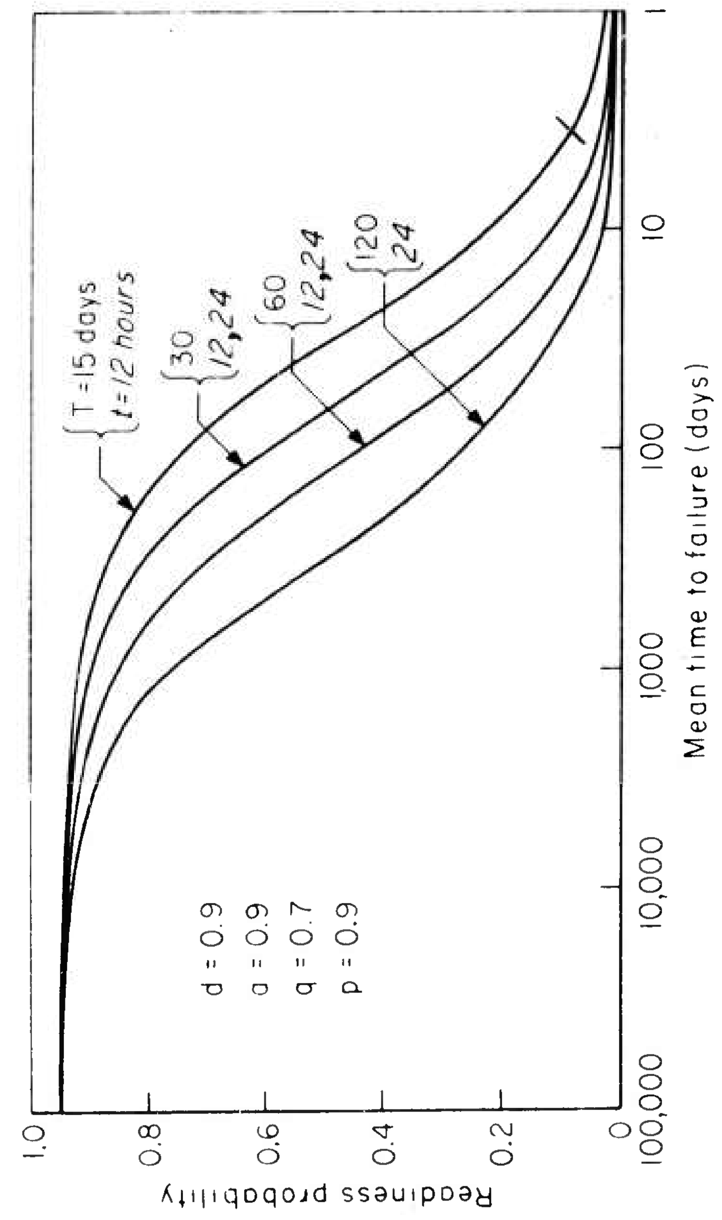


Fig 3—Check periodically

the results was found to be insignificant. In essence, these curves tell you that if you test a function having the characteristics of reliability, and d, using test equipment with the characteristics of p and q, according to a concept with the characteristics of a, t, and T, then a particular readiness will result.

From these sorts of curves we can get the readiness probability for each function on the missile for each combination of method and equipment location. To use these numbers we have laid out the design tableau of Fig. 2. If complete design freedom exists beyond the constraints imposed by safety and physical considerations, then this process will yield the theoretically best readiness-testing program, consistent with the system concept. This program can then serve as a point of departure for physical design studies and necessary operational-design compromises.

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